Interference-Angle Condition, Diffraction and Imagery

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Abstract

The so called coherence-condition contains a geometrical relation, where the angle to (conventionally) light-source has to be smaller than to interval of diffraction-fringes. Here an action of order of radiation does not exist, therefore the name interference-angle condition is offered. In Fraunhofer's manner of observation also inner diffraction-fringes of slit appears outside the focal-plane. At parallel incident light in distance of double focal length with and without optics the same diffraction figures with inner and outer fringes arise, only the figure is inverted with optics. Interference-angle condition and Abbe's theory of imagery complete one another in microscopic imagery.

I. The so called coherence-condition

In order to see diffraction appearances or interference’s as known the so called coherence-condition has to be performed. In figure 1 is drawn schematically an extended light-source and diffraction at double-slit as generally used for derivation of coherence-condition. Θ is the angle from diffraction-slit to light-source and λ the so called wave-length. The coherence-condition reads:

$$X \sin \Theta < \lambda / 2. \qquad (1)$$

On 1865 Verdet [1] it found experimentally at the double-slit that the light-source (nowadays most an illumination-slit) dare not be large as pleasure. Formula (1) was interpreted by wave-theory of light that rays from the edges of light-source dare have at most a phase-difference of λ / 2.

Berge [2] took into consideration also the right side of figure 1, he respected true the diffraction. The first maximum of diffraction-figure at double-slit succeed for sufficient large distances with α as diffraction-angle, d as interval of double-slit, and Y as interval of first diffraction-maxima to

$$d \sin \alpha = \lambda / 2 \qquad (2)$$

With (1):

$$X \sin \Theta < d \sin \alpha \qquad (=\lambda/2) \qquad (3)$$

For small angle is to set:

$$X\, d / 2a < Y\, d / 2b \qquad (= \lambda/2)$$

$$X / a < Y / b \qquad (= \lambda/d) \qquad (4)$$

This formula (4) means in words: "The angle as consequence of diffraction has to be greater than the angle as consequence of geometric extent of light-source" (point-lines in figure 1). The angle in consequence of extent of light-source is transmitted fully to the side of diffraction and caused there a blur in diffraction-figure.

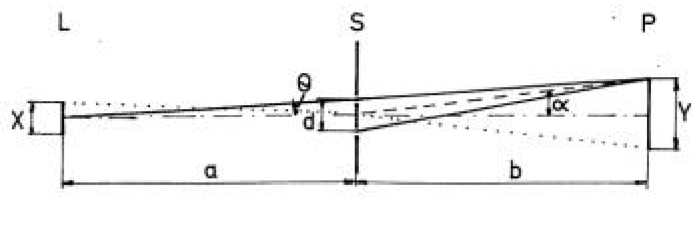

Figure 1. Schematic drawing of light-source and double-slit for derivation of the coherence-condition. L - light-source of the extend X; S - double-slit with the spacing d; P - photoplate with the first diffraction maximum in the spacing Y; Q - half aperture-angle; a -diffraction-angle.

Figure 1 shows the relation in Fresnel's manner of observation without optics. For in Fraunhofer's manner of observation the light-source is imaged in the plane of screen, so the transmission of extend of light-source as blur in the diffraction-figure ensued automatically.

Without to name this as coherence-condition Arkadiew [3] stated that from diffraction-screen the angle to illumination has to be smaller than the angle to interval of diffraction-fringes. Cittert [4], Zernicke [5] and Wawilow [6] established in other connections that the phase of incident light has no influence of the arising interference’s.

Formula (4) is a plain geometric condition and gives no direction on a state of order in radiation that is often concluded from formula (1). Formula (4) would be better denoted as ‘interference-angle-condition’.

Comprehended is to establish that the admissible size of a conventional light-source only describes if interference is generally possible. In formula (3) and (4) frequency falls out only ostensibly, there is presupposed monochromatic light.

II. Discussion of the interference-angle-condition

Our present conception of photons out of spontaneous emission corresponds not at all that photons of an extended conventional light-source should have phase-conditions one with another. Every spontaneous process of emission should be independent of radiation of surroundings. Then at lowest intensities, where only one photon could be in the apparatus, the same diffraction- or interference-figure was found, for example reported by Reynolds, Spartalian and Scarl [7]. According Dirac [8] this is interpreted as interference of photon with itself.
As well there are existing after stimulated emission states of orders and this can designate as coherence.

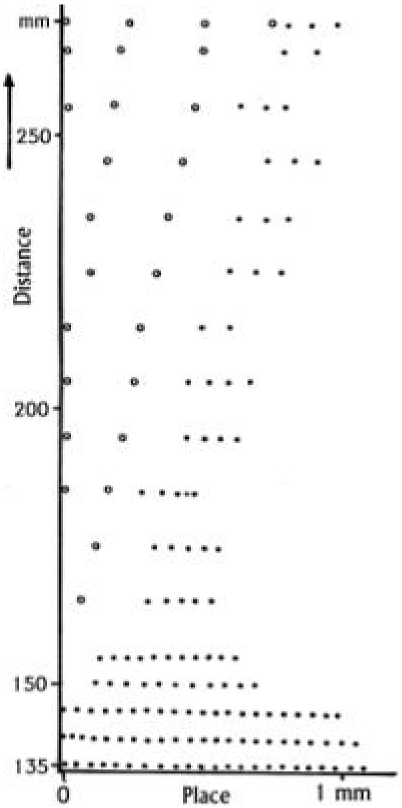

Figure 2. The positions of the maxima of inner and outer diffraction-fringes between single and double focal-length. 135 mm - single focal-length, 270 mm - double focal-length, o maxima of inner fringes, · - outer- fringes. The outermost appearances of inner diffraction-fringes mark the shadow-limit.

According the proposition of Kapitza and Dirac [9] Schwarz [10] showed diffraction-figures with an electron-beam perpendicular to the laser-axis interior of laser. According to that the laser-beam is designated as 'light-crystal' with a lattice-constant in direction of propagation equal to the so called wave-length of light.

The state of order is substantial for stimulated emission. Magar and Mandel [11] found that two lasers can give interference’s without beam-splitter if they were sufficiently stabilised in phase and mode. Richter, Brunner and Paul [12] inferred from this that photons do not only interfere with themselves as Dirac [6] had demanded but also with photons that are in accordance in mode and phase. By this way the large length of interference of special lasers was explained.

The appearances in diffraction, inferred from geometric limitation, shall not be named as coherence because that has nothing to do with states of order of radiation. Consequently, geometric limitations and state of orders are to separate and not to name them commonly as coherence like by Glauber [13] or Vinson [14].

Here is to quote Nieke [15] who proved not only Mach's fringes of the rotating Mach's disk as a physical phenomenon but also showed that Mach's fringes at the limits of half-shade in sunlight are a violation of interference-angle condition, where only is remaining the first maximum of the diffraction-figure of half-plane.

Schrödinger [16] established that an electrically glowing Wollaston-wire with some µm diameter delivers interference's of the double -slit at an angle of 600 with a collecting lens behind the double -slit. The interference-angle -condition was fulfilled for such a thin wire. He quoted Einstein [17] who demanded a directed emission-process. Schrödinger [18] was right when he wrote (translated): "We have true the impression that science is hindered by profound rooted think-customs, some of them are difficult to find out, during others are already disclosed."

III. Diffraction after Fresnel and Fraunhofer

Fresnel [19] used no optics between light-source, diffraction-object and screen as formerly Newton [20] and Young.
In diffraction by Fraunhofer [21] the light-source is imaged with additional lens or objectives on a screen or as intermediate image viewed with an ocular. As result of imaging goes shadow-limits in the image of light-source, and there remains no place for inner fringes. (Compare with Nieke [22]: inner fringes are defined by the position within the shadow-limits). So in Fraunhofer 's manner of observation only outer diffraction-fringes of the slit appears, of course broaden at large images of light-sources if the interference-angle condition is violated. If the image of light-source is larger than the interval of fringes, hardly a diffraction-figure is to evaluate.

In figure 2 the positions of maxima of inner and outer diffraction-fringes are shown between

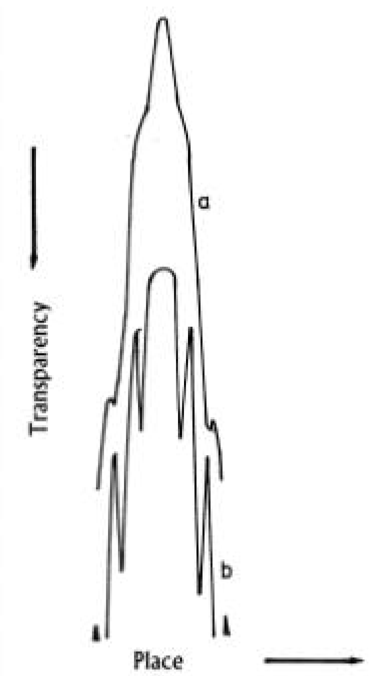

Figure 3. Photometer-curves or diffraction-figures of a slit with a width of 0.6 mm. Illumination-slit 0.001 mm, tessar f' = 135 mm as collimator.
a: without farther optics in 140 mm distance,
b: with a lens f' = 140 mm in 140 mm distance.

single and double focus-plane. A He-Ne laser was focused on the illumination-slit 0.005 mm. In distance of focus length stood a tessar 1:4.5 f' = 135 mm, then followed a second tessar 1:4.5 f' = 135 mm with a slit of 1.5 mm width instead of iris-diaphragm. This slit-width was chosen in order to value easily inner and outer fringes. In the focus-plane the known diffraction-figure of outer fringes appears but already in a distance of 10 % of focus-length behind focus-plane the nought's order splits up in inner fringes. The intervals of outer fringes grow and their numbers become smaller. In intermediate sphere grow the number of inner fringes, the outer fringes remain in a reduced number. Inner fringes are good formed in the double focal-length and outer fringes have about the double intervals as in single focal-length.

It is to establish that in Fraunhofer's manner of observation inner fringes of slit disappear only in the focal-plane, which is also image-plane of the light-source. Outside of the focal-plane inner fringes are visible again.

Diffraction and imagery

In diffraction with imagery there are two special-cases in the focal-plane in Fraunhofer's manner of observation fringes - this are the outer of slit - and in Fresnel's manner of observation in the same distance (of focal-length) outer fringes have 'he same intervals but not the same intensities as figure 3 shows. Natural in Fresnel's manner of observation additional inner fringes can appear which do not exist in Fraunhofer's observation-manner in focal-plane.

The second special-case happen in double focus-plane. The diffraction-object is illuminated with parallel light and the diffraction-figure is caught-up with and without optics in same distance the distance of double focal-length. So appear in both cases the same diffraction-figures with inner and outer fringes, only the figure with optics is inverted corresponding to imagery. This is shown with a triangular-slit as *diffraction-object with different focal-lengths and corresponding distances in figures 4, 5, and 6.*

IV. Interference-angle condition and Abbe's theory of imagery

The interference-angle condition (4) demands that light-source from diffraction-object has to appear in a smaller angle as the interval of diffraction-fringes. Abbe's theory of microscopic imagery demands that besides the zeroth order least the first order of the diffraction-figure of object-structure has

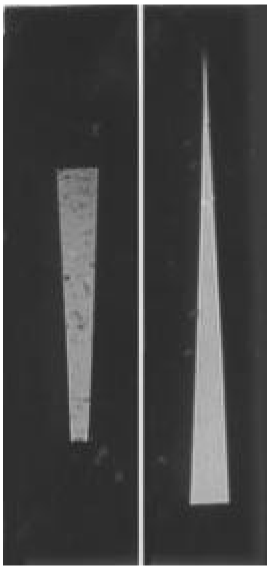

Figure 4. Diffraction-figure of the triangular-slit in double focal-length. A super-pressure mercury-lamp HBO 100 with green-filter and condenser that illuminated a hole-diaphragm 0.1 mm, in 1 m distance stood a lens f' = 1 m, behind that a triangular-slit 0 . . .3 mm that was so parallel illuminated. Behind:

a: Tessar 1 : 2.8, f'= 50 mm, diffraction-figure in 100 mm distance,

b: Objective removed, diffraction-figure in 100 mm distance.

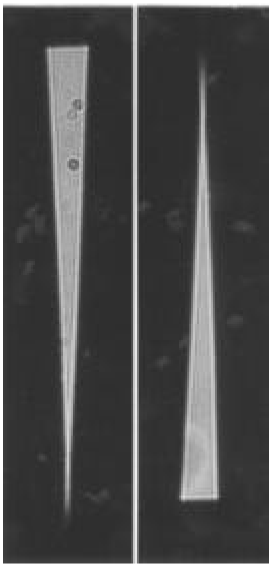

Figure 5. As figure 4.

a: Tessar 1:4.5, f'= 135 mm, diffraction-figure in 270 mm distance,

b: Objective removed, diffraction-figure in 270 mm.

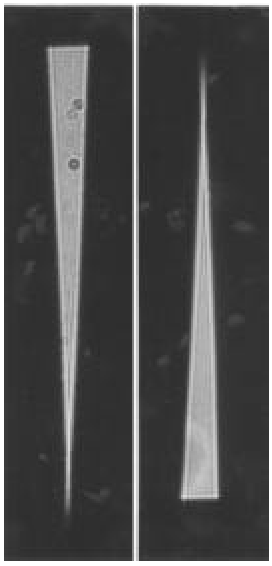

Figure 6. As figure 4.

a: Achromat 1 : 8, ft = 320 mm, diffraction-figure in 640 mm distance.

b: Objective removed, diffraction-figure in 640 mm distance.

to pass the aperture-diaphragm of objective.
The conditions do not harmonize also reference-planes are not identical. However, it is sure that divergence to light-source is carried over image-side in diffraction, this is effective if real diffraction limited the power of resolving. Then the aperture of illumination should be smaller (0.6 -0.8) than the aperture of objective. Already Nieke [23] (there section 7) explained that the diffraction of object should not be strengthen by diffraction at the aperture-diaphragm of microscope objective, for such diffraction

that takes place only near surroundings of a diaphragm. Therefore Abbe's theory and interference-angle condition complete another at microscopic imagery.

Since Zernicke [25] invented phase microscopy, interventions in diffraction-figures were used in the focus-planes of objectives. Zernicke explained this not with coherence but rightly with a sort of schlieren-method. On the contrary for example Yu [26] named all interventions in a diffraction-figure or images of objects as consequence of part-coherence. That is not right as to see in the foregoing sections. Moreover he did not denied conditions of geometry and frequency but he did not distinguish between those with states of order. At interventions in figures of holography a state of order is to expect and here one can speak of coherence. It is to respect that by Pietsch and Menzel [27] Fourier-optics have only a limited validity. Messerschmidt [28] found by experiments at phase-gratings also negative amplitudes of Fourier-analyse and concluded that here the Fourier-analyse has no physical sense. Menzel, Miradé and Weingärtner [29] wrote (translated):". . . that first simplifications of diffraction-theory permits the application of Fourier- theorem.
The cause of this refuse is easy to see: It is not divided between inner and outer diffraction-fringes, and it is not respected that bent light is only coming out of the narrow surroundings of every edge and not from the whole plane. Already Newton [20] III had shown this all in observation 5 and 10.

VI Discussion

The analogy to water-waves suggested former to accept light with order-states or phase-relations. Already Maxwell [30] considered light electro-magnetic disturbance, but he calculated also with waves. With help of interference-angle condition could be shown that there the wave-interpretation of the so called coherence-condition never was necessary for it is explicable pure geometrically.

At diffraction with imagery is to respect that inner and outer diffraction-fringes of slit show another dependence of distance. As generally known grow intervals of outer fringes linear with distance, photons run here rectilinearly. Differently there are the dependence of the inner fringes of slit or diffraction-fringes of the half-plane. Here Fresnel [19] found experimentally an other behaviour. It will do to consider parallel incident light, where intervals of diffraction-fringes grow only with the root of distance (more exactly by Nieke [24]). Newton [20] concluded out of transition from inner to outer fringes at triangular-slits that light-particle have to run eel-like. By Nieke [22], [23] and [24] the shadow-side bent light is displaced shadow-side for it seams to come from the slit-jaw, photons have to run in an S-curve. So it is sure that diffraction and imagery are to describe differently by inner and outer diffraction-fringes. In section 5 are described special-cases.

VIII. Applications

Panarella [31] examined the non-linearity of photo-multipliers at smallest intensity. He used the diffraction-figure of a small hole-diaphragm and he found their fringes blurred by smallest intensity. He diminished the intensity of light with neutral-density-filters that are so arranged in optical-path that they could reduce the interference-angle-condition as result of scattering in this filter. Here-upon could hint the blurred fringes. Jeffers, Wadlinger and Hunter [32] confirmed this result with a small slit instead hole-diaphragm, but they used the same apparatus with diminishing by density-filters. Therefore here is to prove that this was no effect of reducing of interference-angle condition.

References

[1] M. E. Verdet, Ann. Sci. L'École Norm. Super. (Paris) **2** (1865) 291.
[2] E. Berge, Math. naturwiss. Unterricht **27** (1974) 326.
[3] W. Arkadiew, Phys. Z **14** (1913) 832.
[4] P. H. Cittert, Physica **1** (1934) 201.
[5] F. Zernicke, Physica **5** (1938) 785.
[6] S. I. Wawilow, Die Mikrostruktur des Lichtes. Akademie-Verlag, Berlin 1954, S. 62, 76 u. 84.

[7] G. T. Reynolds, K. Spartalian u. D. E. Scarl, Nuovo chim. **61** B (1969) 355.
[8] P. A. M. Dirac, Die Prinzipien der Quantenmechanik. Hirzel, Leipzig 1930.
The Principles of Quantum Mechanics. Clarendon Press Oxford 1935, 1947, 1958.
[9] P. L. Kapitza u. P. A. M. Dirac, Proc. Cambridge Phil. Soc. **28** (1933) 297.
[10] H. Schwarz, Z. Phys. **204** (1967) 276; Phys. Bl. **26** (1970) 436.
[11] G. Magyar u. L. Mandel, Nature **198** (1963) 255.
[12] G. Richter, W. Brunner u. H. Paul, Ann. Physik (7) **14** (1964) 239.
[13] R. J. Glauber, Phys. Rev. **130** (1963) 2529; **131** (1963) 2766.
[14] J. F. Vinson, Optische Kohärenz. WIB 85, Akademie-Verlag Berlin, Pergamon Press, Oxford, Vieweg Braunschweig 1971.
[15] H. Nieke, Exp. Techn. Physik **31** (1983) 53.
[16] E. Schrödinger, Ann. Physik (IV) **61** (1920) 69.
[17] A. Einstein, Phys. Z. **18** (1917) 121.
[18] E. Schrödinger, Nature and the Greeks. Univ. Press Cambridge 1954;
Die Natur und die Griechen. Rowohlt Nr. 28, Hamburg 1956, S. 35.
[19] A. J. Fresnel, Oeuvre Complétes I. Paris 1866;
Abhandlungen über die Beugung des Lichtes. Ostwalds Klassiker Nr. 215, Engelmann, Leipzig 1926.
[20] I. Newton, Opticks, or a Treatise of the Reflexions, Refractions, Inflexions and Colours of Light. London 1704.Opera quae extant omnis, Tom IV, London 1782;
Optik II +III. Übers. W. Abendroth, 0stwald's Klassiker Nr. 97. Engelmann, Leipzig 1898;
Neuauflage Bd 96/97, Vieweg, Braunschweig 1983;
Optique, Trac. J. P. Marat 1787; Bourgois, Paris 1989
[21] J. v. Fraunhofer, Gesammelte Schriften. Verl. bayr. Akad. München 1888.
[22] H. Nieke, Newtons Beugungsexperimente und ihre Weiter
führung. Halle 1997, Comp. Print 1, Arbeit 1
(vorhanden in vielen Deutschen Universitätsbibliotheken)
Newton's Diffraction Experiments and their
Continuation. Halle 1997, comp. print 3, paper 1
(available in some university libraries).
[23] As [22], paper 2.
[24] As [22], paper 3.
[25] F. Zernicke, Physica **9** (1942) 686.
[26] F. T. S. Yu, in Ed. F. Wolf: Progress in Optics XXIII (1986) 222.
[27] K. Pietsch u. E. Menzel, Optik **12** (1955) 203.
[28] W. Messerschmidt, Optik **12** (1955) 297.
[29] E. Menzel, W. Miradé u. I. Weinberger, Fourieroptik und Holographie. Springer, Wien u. New York 1973, quotation in foreword.
[30] J. C. Maxwell, The scientific papers. Cambridge 1890;
Hrsg. L. Boltzmann, Über physikalische Kraftlinien. Ostwalds Klassiker Nr. 102, Engelmann, Leipzig 1898.
[31] E. Panarella, Speculations Sci. Techn. **8** (1985) 35.
[32] S. Jeffers, R. Wadlinger a. G, Hunter, Can. J. Phys. **69** (1991) 1471.